

## p-q-Formel

---

### Aufgabe

Lösen Sie die folgenden quadratischen Gleichungen mit Hilfe der p-q-Formel:

a)  $x^2 - x - 20 = 0$

b)  $x^2 - 15x + 56 = 0$

c)  $x^2 - 2x + 1 = 0$

d)  $x^2 - 12x + 27 = 0$

e)  $x^2 + 20x + 100 = 0$

f)  $x^2 - 10x + 24 = 0$

g)  $x^2 - x - 12 = 0$

h)  $x^2 + 15x + 54 = 0$

i)  $x^2 + 13x + 42 = 0$

j)  $x^2 - 19x + 90 = 0$

k)  $x^2 + 10x + 16 = 0$

l)  $x^2 + x - 30 = 0$

m)  $x^2 - 9x + 20 = 0$

n)  $x^2 + 18x + 80 = 0$

o)  $x^2 + 5x + 4 = 0$

p)  $x^2 - 12x + 36 = 0$

q)  $x^2 + 12x + 32 = 0$

r)  $x^2 - 6x - 7 = 0$

s)  $x^2 - 8x + 12 = 0$

t)  $x^2 - 11x + 24 = 0$

u)  $x^2 + 4x + 4 = 0$

v)  $x^2 + 4x + 3 = 0$

w)  $x^2 - 10x + 0 = 0$

x)  $x^2 - 11x + 28 = 0$

y)  $x^2 + 8x + 7 = 0$

z)  $x^2 - 17x + 70 = 0$

## Rechenweg

a)  $p = -1$   
 $q = -20$

$$\begin{aligned}x_{1,2} &= -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \\&= -\frac{-1}{2} \pm \sqrt{\left(\frac{-1}{2}\right)^2 + 20} \\&= \frac{1}{2} \pm \sqrt{\frac{1}{4} + 20} \\&= \frac{1}{2} \pm \sqrt{\frac{81}{4}} \\&= \frac{1}{2} \pm \frac{9}{2} \\x_1 &= -4 \\x_2 &= 5\end{aligned}$$

c)  $p = -2$   
 $q = 1$

$$\begin{aligned}x_{1,2} &= -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \\&= -\frac{-2}{2} \pm \sqrt{\left(\frac{-2}{2}\right)^2 - 1} \\&= 1 \pm \sqrt{1 - 1} \\&= 1 \pm \sqrt{0} \\&= 1 \pm 0 \\x &= 1\end{aligned}$$

e)  $p = 20$   
 $q = 100$

$$\begin{aligned}x_{1,2} &= -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \\&= -\frac{20}{2} \pm \sqrt{\left(\frac{20}{2}\right)^2 - 100} \\&= -10 \pm \sqrt{100 - 100} \\&= -10 \pm \sqrt{0} \\&= -10 \pm 0 \\x &= -10\end{aligned}$$

b)  $p = -15$   
 $q = 56$

$$\begin{aligned}x_{1,2} &= -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \\&= -\frac{-15}{2} \pm \sqrt{\left(\frac{-15}{2}\right)^2 - 56} \\&= \frac{15}{2} \pm \sqrt{\frac{225}{4} - 56} \\&= \frac{15}{2} \pm \sqrt{\frac{1}{4}} \\&= \frac{15}{2} \pm \frac{1}{2} \\x_1 &= 7 \\x_2 &= 8\end{aligned}$$

d)  $p = -12$   
 $q = 27$

$$\begin{aligned}x_{1,2} &= -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \\&= -\frac{-12}{2} \pm \sqrt{\left(\frac{-12}{2}\right)^2 - 27} \\&= 6 \pm \sqrt{36 - 27} \\&= 6 \pm \sqrt{9} \\&= 6 \pm 3 \\x_1 &= 3 \\x_2 &= 9\end{aligned}$$

f)  $p = -10$   
 $q = 24$

$$\begin{aligned}x_{1,2} &= -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \\&= -\frac{-10}{2} \pm \sqrt{\left(\frac{-10}{2}\right)^2 - 24} \\&= 5 \pm \sqrt{25 - 24} \\&= 5 \pm \sqrt{1} \\&= 5 \pm 1 \\x_1 &= 4 \\x_2 &= 6\end{aligned}$$

g)  $p = -1$   
 $q = -12$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$= -\frac{-1}{2} \pm \sqrt{\left(\frac{-1}{2}\right)^2 + 12}$$

$$= \frac{1}{2} \pm \sqrt{\frac{1}{4} + 12}$$

$$= \frac{1}{2} \pm \sqrt{\frac{49}{4}}$$

$$= \frac{1}{2} \pm \frac{7}{2}$$

$x_1 = -3$   
 $x_2 = 4$

h)  $p = 15$   
 $q = 54$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$= -\frac{15}{2} \pm \sqrt{\left(\frac{15}{2}\right)^2 - 54}$$

$$= -\frac{15}{2} \pm \sqrt{\frac{225}{4} - 54}$$

$$= -\frac{15}{2} \pm \sqrt{\frac{9}{4}}$$

$$= -\frac{15}{2} \pm \frac{3}{2}$$

$x_1 = -9$   
 $x_2 = -6$

i)  $p = 13$   
 $q = 42$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$= -\frac{13}{2} \pm \sqrt{\left(\frac{13}{2}\right)^2 - 42}$$

$$= -\frac{13}{2} \pm \sqrt{\frac{169}{4} - 42}$$

$$= -\frac{13}{2} \pm \sqrt{\frac{1}{4}}$$

$$= -\frac{13}{2} \pm \frac{1}{2}$$

$x_1 = -7$   
 $x_2 = -6$

j)  $p = -19$   
 $q = 90$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$= -\frac{-19}{2} \pm \sqrt{\left(\frac{-19}{2}\right)^2 - 90}$$

$$= \frac{19}{2} \pm \sqrt{\frac{361}{4} - 90}$$

$$= \frac{19}{2} \pm \sqrt{\frac{1}{4}}$$

$$= \frac{19}{2} \pm \frac{1}{2}$$

$x_1 = 9$   
 $x_2 = 10$

k)  $p = 10$   
 $q = 16$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$= -\frac{10}{2} \pm \sqrt{\left(\frac{10}{2}\right)^2 - 16}$$

$$= -5 \pm \sqrt{25 - 16}$$

$$= -5 \pm \sqrt{9}$$

$$= -5 \pm 3$$

$x_1 = -8$   
 $x_2 = -2$

l)  $p = 1$   
 $q = -30$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$= -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + 30}$$

$$= -\frac{1}{2} \pm \sqrt{\frac{1}{4} + 30}$$

$$= -\frac{1}{2} \pm \sqrt{\frac{121}{4}}$$

$$= -\frac{1}{2} \pm \frac{11}{2}$$

$x_1 = -6$   
 $x_2 = 5$

m)  $p = -9$   
 $q = 20$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$= -\frac{-9}{2} \pm \sqrt{\left(\frac{-9}{2}\right)^2 - 20}$$

$$= \frac{9}{2} \pm \sqrt{\frac{81}{4} - 20}$$

$$= \frac{9}{2} \pm \sqrt{\frac{1}{4}}$$

$$= \frac{9}{2} \pm \frac{1}{2}$$

$x_1 = 4$

$x_2 = 5$

o)  $p = 5$   
 $q = 4$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$= -\frac{5}{2} \pm \sqrt{\left(\frac{5}{2}\right)^2 - 4}$$

$$= -\frac{5}{2} \pm \sqrt{\frac{25}{4} - 4}$$

$$= -\frac{5}{2} \pm \sqrt{\frac{9}{4}}$$

$$= -\frac{5}{2} \pm \frac{3}{2}$$

$x_1 = -4$

$x_2 = -1$

q)  $p = 12$   
 $q = 32$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$= -\frac{12}{2} \pm \sqrt{\left(\frac{12}{2}\right)^2 - 32}$$

$$= -6 \pm \sqrt{36 - 32}$$

$$= -6 \pm \sqrt{4}$$

$$= -6 \pm 2$$

$x_1 = -8$

$x_2 = -4$

n)  $p = 18$   
 $q = 80$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$= -\frac{18}{2} \pm \sqrt{\left(\frac{18}{2}\right)^2 - 80}$$

$$= -9 \pm \sqrt{81 - 80}$$

$$= -9 \pm \sqrt{1}$$

$$= -9 \pm 1$$

$x_1 = -10$

$x_2 = -8$

p)  $p = -12$   
 $q = 36$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$= -\frac{-12}{2} \pm \sqrt{\left(\frac{-12}{2}\right)^2 - 36}$$

$$= 6 \pm \sqrt{36 - 36}$$

$$= 6 \pm \sqrt{0}$$

$$= 6 \pm 0$$

$$x = 6$$

r)  $p = -6$   
 $q = -7$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$= -\frac{-6}{2} \pm \sqrt{\left(\frac{-6}{2}\right)^2 + 7}$$

$$= 3 \pm \sqrt{9 + 7}$$

$$= 3 \pm \sqrt{16}$$

$$= 3 \pm 4$$

$x_1 = -1$

$x_2 = 7$

$$\text{s) } \begin{aligned} p &= -8 \\ q &= 12 \end{aligned}$$

$$\begin{aligned} x_{1,2} &= -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \\ &= -\frac{-8}{2} \pm \sqrt{\left(\frac{-8}{2}\right)^2 - 12} \\ &= 4 \pm \sqrt{16 - 12} \\ &= 4 \pm \sqrt{4} \\ &= 4 \pm 2 \\ x_1 &= 2 \\ x_2 &= 6 \end{aligned}$$

$$\text{t) } \begin{aligned} p &= -11 \\ q &= 24 \end{aligned}$$

$$\begin{aligned} x_{1,2} &= -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \\ &= -\frac{-11}{2} \pm \sqrt{\left(\frac{-11}{2}\right)^2 - 24} \\ &= \frac{11}{2} \pm \sqrt{\frac{121}{4} - 24} \\ &= \frac{11}{2} \pm \sqrt{\frac{25}{4}} \\ &= \frac{11}{2} \pm \frac{5}{2} \\ x_1 &= 3 \\ x_2 &= 8 \end{aligned}$$

$$\text{u) } \begin{aligned} p &= 4 \\ q &= 4 \end{aligned}$$

$$\begin{aligned} x_{1,2} &= -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \\ &= -\frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 - 4} \\ &= -2 \pm \sqrt{4 - 4} \\ &= -2 \pm \sqrt{0} \\ &= -2 \pm 0 \\ x &= -2 \end{aligned}$$

$$\text{v) } \begin{aligned} p &= 4 \\ q &= 3 \end{aligned}$$

$$\begin{aligned} x_{1,2} &= -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \\ &= -\frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 - 3} \\ &= -2 \pm \sqrt{4 - 3} \\ &= -2 \pm \sqrt{1} \\ &= -2 \pm 1 \\ x_1 &= -3 \\ x_2 &= -1 \end{aligned}$$

$$\begin{aligned} \text{w)} \quad p &= -10 \\ q &= 0 \end{aligned}$$

$$\begin{aligned} x_{1,2} &= -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \\ &= -\frac{-10}{2} \pm \sqrt{\left(\frac{-10}{2}\right)^2 - 0} \\ &= 5 \pm \sqrt{25 - 0} \\ &= 5 \pm \sqrt{25} \\ &= 5 \pm 5 \\ x_1 &= 0 \\ x_2 &= 10 \end{aligned}$$

$$\begin{aligned} \text{x)} \quad p &= -11 \\ q &= 28 \end{aligned}$$

$$\begin{aligned} x_{1,2} &= -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \\ &= -\frac{-11}{2} \pm \sqrt{\left(\frac{-11}{2}\right)^2 - 28} \\ &= \frac{11}{2} \pm \sqrt{\frac{121}{4} - 28} \\ &= \frac{11}{2} \pm \sqrt{\frac{9}{4}} \\ &= \frac{11}{2} \pm \frac{3}{2} \\ x_1 &= 4 \\ x_2 &= 7 \end{aligned}$$

$$\begin{aligned} \text{y)} \quad p &= 8 \\ q &= 7 \end{aligned}$$

$$\begin{aligned} x_{1,2} &= -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \\ &= -\frac{8}{2} \pm \sqrt{\left(\frac{8}{2}\right)^2 - 7} \\ &= -4 \pm \sqrt{16 - 7} \\ &= -4 \pm \sqrt{9} \\ &= -4 \pm 3 \\ x_1 &= -7 \\ x_2 &= -1 \end{aligned}$$

$$\begin{aligned} \text{z)} \quad p &= -17 \\ q &= 70 \end{aligned}$$

$$\begin{aligned} x_{1,2} &= -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \\ &= -\frac{-17}{2} \pm \sqrt{\left(\frac{-17}{2}\right)^2 - 70} \\ &= \frac{17}{2} \pm \sqrt{\frac{289}{4} - 70} \\ &= \frac{17}{2} \pm \sqrt{\frac{9}{4}} \\ &= \frac{17}{2} \pm \frac{3}{2} \\ x_1 &= 7 \\ x_2 &= 10 \end{aligned}$$

## Lösung

a)  $x_1 = -4$   
 $x_2 = 5$

d)  $x_1 = 3$   
 $x_2 = 9$

g)  $x_1 = -3$   
 $x_2 = 4$

j)  $x_1 = 9$   
 $x_2 = 10$

m)  $x_1 = 4$   
 $x_2 = 5$

p)  $x = 6$

s)  $x_1 = 2$   
 $x_2 = 6$

v)  $x_1 = -3$   
 $x_2 = -1$

y)  $x_1 = -7$   
 $x_2 = -1$

b)  $x_1 = 7$   
 $x_2 = 8$

e)  $x = -10$

h)  $x_1 = -9$   
 $x_2 = -6$

k)  $x_1 = -8$   
 $x_2 = -2$

n)  $x_1 = -10$   
 $x_2 = -8$

q)  $x_1 = -8$   
 $x_2 = -4$

t)  $x_1 = 3$   
 $x_2 = 8$

w)  $x_1 = 0$   
 $x_2 = 10$

z)  $x_1 = 7$   
 $x_2 = 10$

c)  $x = 1$

f)  $x_1 = 4$   
 $x_2 = 6$

i)  $x_1 = -7$   
 $x_2 = -6$

l)  $x_1 = -6$   
 $x_2 = 5$

o)  $x_1 = -4$   
 $x_2 = -1$

r)  $x_1 = -1$   
 $x_2 = 7$

u)  $x = -2$

x)  $x_1 = 4$   
 $x_2 = 7$